PROBLEM ON 2009 OCTOBER 22

MVHS NUMBER THEORY GROUP

Today's problem is the easiest of the week and is thus worth only **1 Point**. Pick any 4-digit integer (call it n) and any 3-digit integer (call it m). You can use any base you like, but 10 will probably be the easiest. It is a fact that if p is a prime that divides both m and n, then p will also divide the difference $n - m$ and the difference $m - n$ (it doesn't matter which comes first). This can be shown as follows. Since p divides m and n, we can write

$$\begin{aligned} m &= p \cdot a \\ n &= p \cdot b \end{aligned}$$

where a, b are other integers. We can then write the difference $m - n$ in terms of p, a, and b and factor out a p using the distributive property.

$$\begin{aligned} m - n &= p \cdot a - p \cdot b \\ &= p \cdot (a - b) \end{aligned}$$

Similarly we have that $n - m = p \cdot (b - a)$. Your task is to use this fact (which we went over on Wednesday) to find the *largest* integer (not necessarily prime) that divides both m and n. For a specific example in the notes we showed that the largest integer dividing both 2145 and 561 is 33. You must say a few words about why your answer is correct.